331 vector-like models with mirror fermions as a possible solution for the discrepancy in the $b-$quark asymmetries, and for the neutrino mass and mixing pattern

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Abstract

A general study of the fermionic structure of the 331 models with β arbitrary shows the possibility of obtaining 331 vector-like models with mirror fermions. On one hand, the existence of mirror fermions gives a possible way to fit the discrepancy in the bottom quark asymmetries from the prediction of the standard model. On the other hand, the vector-like nature of the model permits to address the problem of the fermion mass hierarchy, and in particular the problem of the neutrino mass and mixing pattern. Specifically, we consider a model with four families and $\beta = -1/\sqrt{3}$ where the additional family corresponds to a mirror fermion of the third generation of the Standard Model. We also show how to generate ansatzs about the mass matrices of the fermions according to the phenomenology. In particular, it is possible to get a natural fit for the neutrino hierarchical masses and mixing angles. Moreover, by means of the mixing between the third quark family and its mirror fermion, a possible solution for the A_{FB}^b discrepancy is obtained.

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Keywords: 331 models, mirror fermions, cancellation of anomalies, ansatz for mass matrices, neutrino mixing.

1 Introduction

The models with gauge symmetry $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$, also called 331 models are well motivated models that could address problems such as the origin of families, the hierarchy problem in grand unified theories and the charge quantization problem [1]. Nevertheless, current versions of the 331 model cannot provide an explanation about the mass hierarchy and mixing of the fermions, because models with vector-like multiplets are necessary to explain the family hierarchy. In particular, the neutrinos do not exhibit a strong family hierarchy pattern as it happens with the other fermions; the mixing angles for the atmospheric and solar neutrinos are not small; and the quotient $\left(\delta m_{sun}^2/\delta m_{atm}^2\right)$ is of the order of $0.02-0.03$, these facts suggest to modify the see-saw mechanism in order to cancel the hierarchy in the mass generation for the neutrinos, such modifications are usually implemented by introducing vector-like fermion multiplets [2]. On the other hand, since the traditional 331 models are purely left-handed, they cannot account about parity breaking. Moreover, their left-handed nature along with the weakness of the $Z-Z'$ mixing, prevent such models to explain the deviation of the $b-$quark asymmetry A_b from the value predicted by SM [3]. An interesting alternative to solve this puzzle is to consider models with mirror fermions (MF) that couple with right-handed chirality to the electroweak gauge fields from which the A_b and A_{FB}^b deviations could be fitted [4]. The fitting of this deviation could be achieved by either a modification in the right-handed couplings of Z_μ with the $b-$quark or by modifying the right-handed couplings of the top quark which enter in the correction of the $Zb\overline{b}$ vertex. As a consequence, the $|V_{tb}|$ CKM element could change as well, this fact could give a hint concerning the mass generation mechanism for the ordinary fermions.

According to the discussion above, it would be desirable to get vector-like 331 models with mirror fermions, that might in principle be able to generate right-handed couplings for the bottom and top quarks

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and at the same time provide a possible explanation to the fermion mass hierarchy. A recent study shows the possibility of finding 331 models with such features [5]. The present manuscript describes the minimal 331 vector-like model with MF obtained in Ref. [5] and the way in which such model could solve the problems addressed above.

2 General fermionic structure

In the 331 models, the electric charge is defined as a linear combination of the diagonal generators of the group

$$Q = T_3 + \beta T_8 + XI, \tag{2.1}$$

and the value of the β parameter determines the fermion assignment and the electric charges of the exotic spectrum [1, 6]. Hence, such parameter is used to classify the different 331 models. An analysis of the fermion representations shows that the left-handed multiplets lie in either the **3** or $\mathbf{3}^*$ representations of $SU(3)_L$.

Ref. [5] has studied the possible fermionic structures for 331 models with β arbitrary based on cancellation of anomalies and demanding a fermionic spectrum with a minimal number of exotic particles. Denoting N as the number of leptonic generations and M the number of quark generations, minimization of the exotic fermionic spectrum requires to associate only one lepton and one quark $SU(3)_L$ multiplet with each generation, and at most one right-handed singlet associated with each left-handed fermion. From such assumptions we obtain the fermionic spectrum (containing the SM spectrum) displayed in table 1 for the quarks and leptons.

It is possible to have in a single model any number of left-handed multiplets in either the **3** or $\mathbf{3}^*$ representations. In the most general case, each multiplet can transform as

$$\begin{cases} q_L^{(m)}, q_L^{(m^*)} : \; m = \underbrace{1, 2, \ldots, k}_{3k \; triplets}; \; m^* = \underbrace{k+1, k+2, \ldots, M}_{3(M-k) \; antitriplets} \\ \ell_L^{(n)}, \ell_L^{(n^*)} \quad : \; n = \underbrace{1, 2, \ldots, j}_{j \; triplets}; \; n^* = \underbrace{j+1, j+2, \ldots, N}_{N-j \; antitriplets} \end{cases} \tag{2.2}$$

where the first $3k$-th multiplets of quarks lie in the **3** representation while the latter $3(M-k)$ lie in the $\mathbf{3}^*$ representation for a total of $3M$ quark left-handed multiplets. The factor 3 in the number of quark left-handed multiplets owes to the existence of three colors. Similarly the first j left-handed multiplets of leptons are taken in the representation **3** and the latter $(N-j)$ are taken in the $\mathbf{3}^*$ representation, for a total of N leptonic left-handed multiplets. Table 2 shows the possible multiplet structures in the fermionic sector compatible with cancellation of anomalies with β arbitrary [5]. Representations with $N = 1$ are forbidden.

Furthermore, the requirement for the model to be $SU(3)_c$ vector-like demands the presence of right-handed quark singlets, while right-handed neutral lepton singlets are optional. The possible charged right-handed leptonic singlet structures compatible with cancellation of anomalies are also studied in Ref. [5]. Tables 3, 4 summarize the posssible leptonic singlet structures and the solutions compatible with cancellation of anomalies; in that tables we use $\Theta_{e^{(1)}}, \Theta_{E^{(1)}}$ to denote the number of singlets lying in the **3** representation associated with the ordinary and exotic leptons repectively; while $\Theta_{e^{(j+1)}} \Theta_{E^{(j+1)}}$ stands for the corresponding singlets in the $\mathbf{3}^*$ representation[1].

3 331 vector-like models with MF

We can see from table 2 that models with $N = 2, 4$ are $SU(3)_L$ vector-like with respect to the quark and lepton sectors separately. Besides, one of the three possible structures of fermionic multiplets with $N = 6$ is also vector-like in quark and leptons sector. Taking the minimal fermionic content that could include the SM fermions, we shall study the case of $N = 4$. Finally, if we demand for the model not to have exotic charges we are lead to only two values of the β parameter i.e. $\beta = \pm 1/\sqrt{3}$. According to tables 3, 4 we see that the only vector-like structure with no exotic charges (i.e. $\beta = \pm 1/\sqrt{3}$) is the one described by the

[1] It worths remarking that $\Theta_l = 0, 1$ because we have assumed that at most one singlet is associated with each fermion multiplet.

Quarks	Q_ψ	X_ψ
$q_L^{(m)} = \begin{pmatrix} U^{(m)} \\ D^{(m)} \\ J^{(m)} \end{pmatrix}_L : \mathbf{3}$	$\begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \\ \frac{1}{6}-\frac{\sqrt{3}\beta}{2} \end{pmatrix}$	$X^L_{q^{(m)}} = \frac{1}{6} - \frac{\beta}{2\sqrt{3}}$
$U_R^{(m)} : \mathbf{1}$ $D_R^{(m)} : \mathbf{1}$ $J_R^{(m)} : \mathbf{1}$	$\frac{2}{3}$ $-\frac{1}{3}$ $\frac{1}{6}-\frac{\sqrt{3}\beta}{2}$	$X^R_{U^{(m)}} = \frac{2}{3}$ $X^R_{D^{(m)}} = -\frac{1}{3}$ $X^R_{J^{(m)}} = \frac{1}{6} - \frac{\sqrt{3}\beta}{2}$
$q_L^{(m^*)} = \begin{pmatrix} D^{(m^*)} \\ -U^{(m^*)} \\ J^{(m^*)} \end{pmatrix}_L : \mathbf{3}^*$	$\begin{pmatrix} -\frac{1}{3} \\ \frac{2}{3} \\ \frac{1}{6}+\frac{\sqrt{3}\beta}{2} \end{pmatrix}$	$X^L_{q^{(m^*)}} = -\frac{1}{6} - \frac{\beta}{2\sqrt{3}}$
$D_R^{(m^*)} : \mathbf{1}$ $U_R^{(m^*)} : \mathbf{1}$ $J_R^{(m^*)} : \mathbf{1}$	$-\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{6}+\frac{\sqrt{3}\beta}{2}$	$X^R_{D^{(m^*)}} = -\frac{1}{3}$ $X^R_{U^{(m^*)}} = \frac{2}{3}$ $X^R_{J^{(m^*)}} = \frac{1}{6} + \frac{\sqrt{3}\beta}{2}$
Leptons	Q_ψ	X_ψ
$\ell_L^{(n)} = \begin{pmatrix} \nu^{(n)} \\ e^{(n)} \\ E^{(n)} \end{pmatrix}_L : \mathbf{3}$	$\begin{pmatrix} 0 \\ -1 \\ -\frac{1}{2}-\frac{\sqrt{3}\beta}{2} \end{pmatrix}$	$X^L_{\ell^{(n)}} = -\frac{1}{2} - \frac{\beta}{2\sqrt{3}}$
$\nu_R^{(n)} : \mathbf{1}$ $e_R^{(n)} : \mathbf{1}$ $E_R^{(n)} : \mathbf{1}$	0 -1 $-\frac{1}{2}-\frac{\sqrt{3}\beta}{2}$	$X^R_{\nu^{(n)}} = 0$ $X^R_{e^{(n)}} = -1$ $X^R_{E^{(n)}} = -\frac{1}{2} - \frac{\sqrt{3}\beta}{2}$
$\ell_L^{(n^*)} = \begin{pmatrix} e^{(n^*)} \\ -\nu^{(n^*)} \\ E^{(n^*)} \end{pmatrix}_L : \mathbf{3}^*$	$\begin{pmatrix} -1 \\ 0 \\ -\frac{1}{2}+\frac{\sqrt{3}\beta}{2} \end{pmatrix}$	$X^L_{\ell^{(n^*)}} = \frac{1}{2} - \frac{\beta}{2\sqrt{3}}$
$e_R^{(n^*)} : \mathbf{1}$ $\nu_R^{(n^*)} : \mathbf{1}$ $E_R^{(n^*)} : \mathbf{1}$	-1 0 $-\frac{1}{2}+\frac{\sqrt{3}\beta}{2}$	$X^R_{e^{(n^*)}} = -1$ $X^R_{\nu^{(n^*)}} = 0$ $X^R_{E^{(n^*)}} = -\frac{1}{2} + \frac{\sqrt{3}\beta}{2}$

Table 1: *Fermionic content of $SU(3)_L \otimes U(1)_X$ obtained by requiring only one lepton and one quark $SU(3)_L$ multiplet for each generation, and no more than one right-handed singlet for each right-handed field. The structure of left-handed multiplets is the one shown in Eq. (2.2). m and n label the quark and lepton left-handed triplets respectively, while m^*, n^* label the antitriplets, see Eq. (2.2).*

N	Allowed representations		
2	$\ell^{(1)}:3$ $\ell^{(2)}:3^*$ $q^{(1)}:3$ $q^{(2)}:3^*$		
3	$\ell^{(1)},\ell^{(2)},\ell^{(3)}:3^*$ $q^{(1)},q^{(2)}:3$ $q^{(3)}:3^*$	$\ell^{(1)},\ell^{(2)},\ell^{(3)}:3$ $q^{(3)}:3$ $q^{(1)},q^{(2)}:3^*$	
4	$\ell^{(1)},\ell^{(2)}:3$ $\ell^{(3)},\ell^{(4)}:3^*$ $q^{(1)},q^{(2)}:3$ $q^{(3)},q^{(4)}:3^*$		
5	$\ell^{(5)}:3$ $\ell^{(1)},\ell^{(2)},\ell^{(3)},\ell^{(4)}:3^*$ $q^{(3)},q^{(4)},q^{(5)}:3$ $q^{(1)},q^{(2)}:3^*$	$\ell^{(1)},\ell^{(2)},\ell^{(3)},\ell^{(4)}:3$ $\ell^{(5)}:3^*$ $q^{(1)},q^{(2)}:3$ $q^{(3)},q^{(4)},q^{(5)}:3^*$	
6	$\ell^{(1)},\ell^{(2)},\ell^{(3)},$ $\ell^{(4)},\ell^{(5)},\ell^{(6)}\ :3^*$ $q^{(1)},q^{(2)},q^{(5)},q^{(6)}:3$ $q^{(3)},q^{(4)}:3^*$	$\ell^{(1)},\ell^{(2)},\ell^{(3)},$ $\ell^{(4)},\ell^{(5)},\ell^{(6)}\ :3$ $q^{(3)},q^{(4)}:3$ $q^{(1)},q^{(2)},q^{(5)},q^{(6)}:3^*$	$\ell^{(1)},\ell^{(2)},\ell^{(3)}:3$ $\ell^{(4)},\ell^{(5)},\ell^{(6)}:3^*$ $q^{(1)},q^{(2)},q^{(5)}:3$ $q^{(3)},q^{(4)},q^{(6)}:3^*$

Table 2: *Possible representations for the fermion left-handed multiplets compatible with cancellation of anomalies. Each value of $q^{(i)}$ represents three left-handed quark multiplets because of the color factor.*

$\Theta_{e^{(1)}}$	$\Theta_{E^{(1)}}$	$\Theta_{e^{(j+1)}}$	$\Theta_{E^{(j+1)}}$	Solution
1	0	0	1	$\beta=-\sqrt{3}$
0	1	1	0	$\beta=\sqrt{3}$

Table 3: *Solutions for $N=2j=2k\geq 2$*

$\Theta_{e^{(1)}}$	$\Theta_{E^{(1)}}$	$\Theta_{e^{(j+1)}}$	$\Theta_{E^{(j+1)}}$	Solution
0	0	0	0	$\beta=\sqrt{3};\ j=0$ $\beta=-\sqrt{3};\ j=N$
0	0	1	1	$\beta=-\sqrt{3};\ \forall\ j\neq 0$ $\forall\ \beta;\ j=0$
1	1	0	0	$\beta=\sqrt{3};\ \forall\ j\neq N$ $\forall\ \beta;\ j=N$
0	1	1	1	$j=0,\ \forall N,\ \beta=-1/\sqrt{3}$
1	1	0	1	$j=N,\ \beta=1/\sqrt{3}$
1	1	1	1	$\forall\beta,\forall N,\ j\neq 0,N$ if $j=0\Rightarrow\beta=-\sqrt{3}$ if $j=N\Rightarrow\beta=\sqrt{3}$

Table 4: *Solutions for $N=\frac{j+3k}{2}\geq 2$, $0\leq k\leq N$*

first solution in the last row of table (4); in which exactly one right-handed singlet is associated with each leptonic multiplet.

In particular, we shall examine the model with $N = 4$ and $\beta = -1/\sqrt{3}$, where three families refer to the generations at low energies and the other is a mirror family. This is a $SU(3)_L$ vector-like model that has two multiplets in the **3** representation and two multiplets in the **3*** representation in both the quark and lepton sectors. This extension of the 331 model is not reduced to the known models with $\beta = -1/\sqrt{3}$ [7], because in such model the leptons are in three 3-dimensional multiplets. From the phenomenological point of view at low energies, the difference would be in generating ansatz for the mass matrices in the lepton and quark sectors. As we mentioned above, models with vector-like multiplets are necessary to explain the family hierarchy, and mirror fermions are a possible source to solve the deviation of the bottom quark asymmetries from the SM prediction.

4 Model with $N = 4$ and $\beta = -\frac{1}{\sqrt{3}}$

Quarks	Q_ψ	X_ψ
$q_L^{(m)} = \begin{pmatrix} u^{(m)} \\ d^{(m)} \\ J^{(m)} \end{pmatrix}_L : 3$	$\begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \\ \frac{2}{3} \end{pmatrix}$	$X^L_{q^{(m)}} = \frac{1}{3}$
$u_R^{(m)},\, d_R^{(m)},\, J_R^{(m)} : 1$	$\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$	$X^R_{q^{(m)}} = Q_{q^{(m)}}$
$q_L^{(3^*)} = \begin{pmatrix} d^{3^*} \\ -u^{3^*} \\ J^{3^*} \end{pmatrix}_L : 3^*$	$\begin{pmatrix} -\frac{1}{3} \\ \frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$	$X^L_{q^{3^*}} = 0$
$d_R^{3^*},\, u_R^{3^*},\, J_R^{3^*} : 1$	$-\frac{1}{3}, \frac{2}{3}, -\frac{1}{3}$	$X^R_{q^{3^*}} = Q_{q^{3^*}}$
$q_L^{4^*} = \begin{pmatrix} \widetilde{u}^c \\ \widetilde{d}^c \\ \widetilde{J}^c \end{pmatrix}_L : 3^*$	$\begin{pmatrix} -\frac{2}{3} \\ \frac{1}{3} \\ -\frac{2}{3} \end{pmatrix}$	$X^L_{q^{4^*}} = \frac{1}{3}$
$\widetilde{u}^c_R,\, \widetilde{d}^c_R,\, \widetilde{J}^c_R : 1$	$-\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}$	$X^R_{q^{4^*}} = Q_{q^{4^*}}$
Leptons	Q_ψ	X_ψ
$\ell_L^{(n)} = \begin{pmatrix} \nu^{(n)} \\ e^{(n)} \\ N^{0(n)} \end{pmatrix}_L : 3$	$\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$	$X^L_{\ell^{(n)}} = -\frac{1}{3}$
$\nu_R^{(n)}, e_R^{(n)} : 1$	$0, -1, 0$	$X^R_{\ell^{(n)}} = Q_{\ell^{(n)}}$
$\ell_L^{3^*} = \begin{pmatrix} e^{3^*} \\ -\nu^{3^*} \\ E^{3^*-} \end{pmatrix}_L : 3^*$	$\begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix}$	$X^L_{\ell^{3^*}} = \frac{2}{3}$
$e_R^{3^*}, \nu_R^{3^*}, E_R^{3^*-} : 1$	$-1, 0, -1$	$X^R_{\ell^{3^*}} = Q_{\ell^{3^*}}$
$\ell_L^{4^*} = \begin{pmatrix} \widetilde{\nu}^c \\ \widetilde{e}^c \\ \widetilde{N}^{0c} \end{pmatrix}_L : 3^*$	$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$	$X^L_{\ell^{4^*}} = -\frac{1}{3}$
$\widetilde{\nu}^c_R, \widetilde{e}^c_R : 1$	$0, 1, 0$	$X^R_{\ell^{4^*}} = Q_{\ell^{4^*}}$

Table 5: *Fermionic content of $SU(3)_L \otimes U(1)_X$, with $N = 4$,* and $m, n = 1, 2$*. The 4th families which are in the* **3*** *representation, are the mirror fermions of one of the families in the* **3** *representation.*

We consider a model with $\beta = -1/\sqrt{3}$ which is similar to the model described in Ref. [7] at low energies due to the electromagnetic charged assigned to different multiplets. However, this model is not the same as the one in Ref. [7] because the multiplets structure for the quark sector is $SU(3)_C \otimes SU(3)_L$ vector-like, and the leptonic part is not neccesary to cancel the quark anomalies. The leptonic multiplets are also vector-like and anomaly free (see table 5). In the models described in the literature, the quarks anomalies are cancelled out with the leptonic anomalies. In the model with $N = 4$ and $\beta = -1/\sqrt{3}$ there are two 3-multiplets for leptons and two 3-multiplets for quarks and they generate the two heavy families of the SM. Two 3^*-multiplets for quarks and leptons correspond to the first SM family; and the other two 3^*, q_L^{4*} and l_L^{4*}, correspond to a mirror fermion family of the third SM family. So with this assignment, it is possible to get mixing between the bottom quark and its mirror quark d^c in order to modify the right-handed coupling of the bottom quark with the Z gauge boson which in turn might explain the asymmetry deviations A^b and A_{FB}^b [4]. Such discrepancy cannot be explained by a model with only left-handed multiplets such as the SM [8] or the traditional 331 models [3]. The mixing in the mass matrix between the b quark and its mirror fermion permits a solution because the mirror couples with right-handed chirality to the Z_μ gauge field of the SM. On the other hand, the mirror fermions in the leptonic sector are useful to build up ansatz about mass matrices in the neutrino and charged sectors. For the neutrinos corresponding to $SU(2)_L$ doublets, right handed neutrino singlets are introduced to generate masses of Dirac type.

As for the scalar spectrum, three types of representations are considered. The three minimal triplets (whose VEV are shown in table 6) that assure the spontaneous symmetry breaking (SSB) 331 → 321 → 31, and the masses for the gauge fields. Further, an additional scalar in the adjoint representation is included. Such multiplet permits a mixing of the mirror fermions with the ordinary fermions of the SM in order to generate different ansatz for masses. The adjoint representation acquires the VEV's displayed in table 6. Finally, a sextet representation can also be introduced as shown in table 6, it acquires very small VEV's compared with the VEV's of the 331 and electroweak scales ν_χ, ν_ρ and ν_η since they belong to triplet components of $SU(2)_L$ and would not break the relation for $\Delta\rho$. They also permit to generate majorana masses for neutrinos.

$\langle\chi\rangle_0$	$\begin{pmatrix} 0 & 0 & \nu_\chi \end{pmatrix}^T$	$X_\chi = -1/3$
$\langle\rho\rangle_0$	$\begin{pmatrix} 0 & \nu_\rho & 0 \end{pmatrix}^T$	$X_\rho = 2/3$
$\langle\eta\rangle_0$	$\begin{pmatrix} \nu_\eta & 0 & 0 \end{pmatrix}^T$	$X_\eta = 2/3$
$\langle\phi\rangle_0$	$\nu_\chi diag \begin{pmatrix} 1 & 1 & -2 \end{pmatrix}$	$X_\chi = 0$
$\langle S^{ij}\rangle_0$	$V\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$X_S = -1/3$

Table 6: *Scalar sector with* $N = 4$ *and its VEV's.* χ, ρ, η *are triplets in the* **3** representation, ϕ is a multiplet in the adjoint representation, and S lies in the sextet representation. ν_χ *is of the order of the first symmetry breaking.* ν_ρ, ν_η *are of the order of the electroweak scale.* V *is much lower than the electroweak VEV.*

4.1 Mass matrix for quarks

The Yukawa Lagrangian for quarks has the form

$$
\begin{aligned}
\mathcal{L}_Y^q \;=\; & \sum_{\Phi}\sum_{sing.}\sum_{m,m'=1}^{2} h_{q_R}^{m\varphi}\overline{q_L^{(m)}}q_R\Phi \\
& +\frac{1}{2}\overline{q_L^{i(m)}}\left(q_L^{j(m')}\right)^c\left[h_{\Phi}^{mm'}\varepsilon^{ijk}\Phi_k + h_S^{mm'}S^{ij}\right] + h_{q_R}^{3\Phi}\overline{q_L^{(3^*)}}q_R\Phi^* + h_{q_R}^{4\Phi}\overline{q_L^{(4^*)}}q_R\Phi^* \\
& +\frac{1}{2}\overline{q_{iL}^{(3^*)}}\left(q_{jL}^{(3^*)}\right)^c\left[Y_{\Phi}^{33}\varepsilon_{ijk}\Phi^k + Y_S^{33}S_{ij}\right] + \frac{1}{2}\overline{q_{iL}^{(4^*)}}\left(q_{jL}^{(4^*)}\right)^c\left[Y_{\Phi}^{44}\varepsilon_{ijk}\Phi^k + h_S^{44}S_{ij}\right] \\
& +\frac{1}{2}\overline{q_{iL}^{(3^*)}}\left(q_{jL}^{(4^*)}\right)^c\left[Y_{\Phi}^{34}\varepsilon_{ijk}\Phi^k + Y_S^{34}S_{ij}\right] + \frac{1}{2}\overline{q_{iL}^{(4^*)}}\left(q_{jL}^{(3^*)}\right)^c\left[Y_{\Phi}^{43}\varepsilon_{ijk}\Phi^k + Y_S^{43}S_{ij}\right] \\
& +\frac{1}{2}h_{\phi}^{n3}\overline{q_L^{i(n)}}\left(q_{jL}^{(3^*)}\right)^c\phi_j^i + \frac{1}{2}h_{\phi}^{3n}\overline{q_{iL}^{(3^*)}}\left(q_L^{j(n)}\right)^c\phi_i^j \\
& +\frac{1}{2}h_{\phi}^{n4}\overline{q_L^{i(n)}}\left(q_{jL}^{(4^*)}\right)^c\phi_j^i + \frac{1}{2}h_{\phi}^{4n}\overline{q_{iL}^{(4^*)}}\left(q_L^{j(n)}\right)^c\phi_i^j \;+h.c,
\end{aligned} \tag{4.1}
$$

with Φ being any of the η, ρ, χ multiplets, while ϕ, and S correspond to the scalar adjoint and the sextet representation of $SU(3)_L$ respectively. The third and fourth families are written explicitly, since the fourth one correspond to a mirror fermion. The constants $h_{\Phi}^{mm'}$ and Y_{Φ}^{34} are antisymmetric. It should be noted that all possible terms with scalar triplets, adjoints, and sextets are involved. When we take the VEV's from table 6, the mass matrices are obtained.

For the mixing among up-type quarks in the basis $(u_{3^*}, u_1, u_2, \widetilde{u}, J_1, J_2, \widetilde{J})$ we get

$$
M^{up} = \begin{pmatrix} \mathcal{M}_U & \mathcal{M}_{JU} \\ \mathcal{M}_{UJ} & \mathcal{M}_J \end{pmatrix}, \tag{4.2}
$$

where

$$
\mathcal{M}_U = \begin{pmatrix} \nu_\rho h_{u_3}^{3\rho} & \nu_\rho h_{u_1}^{3\rho} & \nu_\rho h_{u_2}^{3\rho} & h_\chi^{43}\nu_\chi \\ \nu_\eta h_{u_3}^{1\eta} & \nu_\eta h_{u_1}^{1\eta} & \nu_\eta h_{u_2}^{1\eta} & h_\phi^{14}\nu_\chi \\ \nu_\eta h_{u_3}^{2\eta} & \nu_\eta h_{u_1}^{2\eta} & \nu_\eta h_{u_2}^{2\eta} & h_\phi^{24}\nu_\chi \\ 0 & 0 & 0 & \nu_\eta h_{\widetilde{u}}^{4\eta} \end{pmatrix}, \quad \mathcal{M}_J = \begin{pmatrix} \nu_\chi h_{J_1}^{1\varkappa} & \nu_\chi h_{J_2}^{1\varkappa} & -2h_\phi^{14}\nu_\chi \\ \nu_\chi h_{J_1}^{2\varkappa} & \nu_\chi h_{J_2}^{2\varkappa} & -2h_\phi^{24}\nu_\chi \\ 0 & 0 & \nu_\chi h_{\widetilde{J}}^{4\varkappa} \end{pmatrix},
$$

$$
\mathcal{M}_{UJ} = \begin{pmatrix} \nu_\chi h_{u_3}^{1\varkappa} & \nu_\chi h_{u_1}^{1\varkappa} & \nu_\chi h_{u_2}^{1\varkappa} & 0 \\ \nu_\chi h_{u_3}^{2\varkappa} & \nu_\chi h_{u_1}^{2\varkappa} & \nu_\chi h_{u_2}^{2\varkappa} & 0 \\ 0 & 0 & 0 & \nu_\eta h_{\widetilde{J}}^{4\eta} \end{pmatrix}, \quad \mathcal{M}_{JU} = \begin{pmatrix} \nu_\rho h_{J_1}^{3\rho} & \nu_\rho h_{J_2}^{3\rho} & h_\eta^{34}\nu_{\eta_1} \\ \nu_{\eta_1} h_{J_1}^{1\eta} & \nu_{\eta_1} h_{J_2}^{1\eta} & 0 \\ \nu_{\eta_1} h_{J_1}^{2\eta} & \nu_{\eta_1} h_{J_2}^{2\eta} & 0 \\ 0 & 0 & \nu_\chi h_{\widetilde{u}}^{4\chi} \end{pmatrix}
$$

and (u_{3^*}, u_1, u_2) correspond to the three families of the SM, $\widetilde{u}$ refers to the mirror fermion of either u_1 or u_2, and $J_1, J_2, \widetilde{J}$ are the exotic quarks with 2/3 electromagnetic charge.

For down-type quarks in the basis $(d_{3^*}, d_1, d_2, \widetilde{d}, J_{3^*})$, the mass matrix yields

$$
M^{down} = \begin{pmatrix} \nu_\eta h_{d_3}^{3\eta} & \nu_\eta h_{d_1}^{3\eta} & \nu_\eta h_{d_2}^{3\eta} & Y_\chi^{34}\nu_\chi & \nu_\eta h_{J_3}^{3\eta} \\ \nu_\rho h_{d_3}^{1\rho} & \nu_\rho h_{d_1}^{1\rho} & \nu_\rho h_{d_2}^{1\rho} & h_\phi^{14}\nu_\chi & \nu_\rho h_{J_3}^{1\rho} \\ \nu_\rho h_{d_3}^{2\rho} & \nu_\rho h_{d_1}^{2\rho} & \nu_\rho h_{d_2}^{2\rho} & h_\phi^{24}\nu_\chi & \nu_\rho h_{J_3}^{2\rho} \\ 0 & 0 & 0 & \nu_\rho Y_{\widetilde{d}}^{4\rho} & 0 \\ \nu_\chi h_{d_3}^{3\varkappa} & \nu_\chi h_{d_1}^{3\varkappa} & \nu_\chi h_{d_2}^{3\varkappa} & Y_\eta^{43}\nu_\eta & \nu_\chi h_{J_3}^{3\varkappa} \end{pmatrix} \tag{4.3}
$$

(d_{3^*}, d_1, d_2) are associated with the three SM families, $\widetilde{d}$ is a down-type mirror quark of either $d_1$or d_2, and J_{3^*} is an exotic down-type quark. When the adjoint representation of the scalar fields is not taken into account, the mixing between $q^{(m)}$ and the quark mirrors $q^{(4^*)}$ does not appear. Such mixing is important to

change the right-handed coupling of the $b-$quark with the Z_μ gauge field, and look for a possible solution for the deviation of the assymmetries A_b and A_{FB}^b of the SM with respect to the experimental data. If the mixing with the mirror quarks were withdrawn and the exotic particles were decoupled, the mirror quarks would acquire masses of the order of the electroweak scale $\nu_\rho h_{\widetilde{d}}^{4\rho}$, $\nu_\eta h_{\widetilde{u}}^{4\rho}$ for the up and down sectors, respectively.

4.2 Mass matrix for Leptons

The Yukawa Lagrangian for leptons keeps the general form shown in Eq. (4.1) for the quarks. However, majorana terms could arise because of the existence of neutral fields. By taking the whole spectrum including right-handed neutrino singlets, Dirac terms are obtained for the charged sector while Dirac and majorana terms appear in the neutral sector.

By including all the possible structures of VEV's, the charged sector in the basis $(e_{3^*}, e_1, e_2, \widetilde{e}, E_{3^*}^-)$ has the following form

$$M^{\ell\pm} = \begin{pmatrix} \nu_\eta h_{e_3}^{3\eta} & \nu_\eta h_{e_1}^{3\eta} & \nu_\eta h_{e_2}^{3\eta} & h_\chi^{34}\nu_\chi & \nu_\eta h_{J_3}^{3\eta} \\ \nu_\rho h_{e_3}^{1\rho} & \nu_\rho h_{e_1}^{1\rho} & \nu_\rho h_{e_2}^{1\rho} & h_\phi^{14}\nu_\chi & \nu_\rho h_{J_3}^{1\rho} \\ \nu_\rho h_{e_3}^{2\rho} & \nu_\rho h_{e_1}^{2\rho} & \nu_\rho h_{e_2}^{2\rho} & h_\phi^{24}\nu_\chi & \nu_\rho h_{J_3}^{2\rho} \\ 0 & 0 & 0 & \nu_\rho h_{\widetilde{e}}^{4\rho} & 0 \\ \nu_\chi h_{e_3}^{3\varkappa} & \nu_\chi h_{e_1}^{3\varkappa} & \nu_\chi h_{e_2}^{3\varkappa} & h_\eta^{43}\nu_{\eta_1} & \nu_\chi h_{E_3}^{3\varkappa} \end{pmatrix}$$

the three first components e_i correspond to the ordinary leptons of the SM, $\widetilde{e}$ is a mirror lepton of e_1 or e_2, and E_{3^*} is an exotic lepton. Like in the case of the quark sector, direct mixings are gotten between all the fields $\ell^{(n)}, \ell^{3^*}$ and the mirrors ℓ^{4^*} by means of the scalars χ, ρ, η and the adjoint ϕ. The mass matrix of charged leptons is similar to the mass matrix of the down-type quarks.

For the neutral lepton sector, we take the following basis of fields

$$\begin{aligned} \psi_L^0 &= \left(\nu_{3L}, \nu_{1L}, \nu_{2L}, (\widetilde{\nu}_R)^c, N_{1L}^0, N_{2L}^0, \left(\widetilde{N}_R^0\right)^c\right)^T, \\ \psi_R^0 &= \left(\nu_{3R}, \nu_{1R}, \nu_{2R}, (\widetilde{\nu}_L)^c\right)^T, \end{aligned} \tag{4.4}$$

where ν_{iL} are the SM fields, ν_{iR} are sterile neutrinos and the right-handed components of SM neutrinos. With these components the Dirac mass matrix is constructed like the up quarks mass matrix; $(\widetilde{\nu}_{L,R})^c$ are mirror fermions, and N_{iL}^0 are exotic neutral fermions. The mass terms are written as

$$\mathfrak{L}_Y^0 = \left(\overline{\psi_L^0}; \overline{(\psi_R^0)^c}\right) \begin{pmatrix} M_L & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} (\psi_L^0)^c \\ \psi_R^0 \end{pmatrix} + h.c, \tag{4.5}$$

where very massive majorana terms M_R have been introduced between the singlets ψ_R^0, corresponding to sterile neutrinos with right-handed chirality. We shall suppose that in this basis the mass matrix M_R is diagonal. Such terms can be introduced without a SSB because they are $SU(3)_L \otimes U(1)_X$ invariant. Besides, they correspond to heavy majorana mass terms for the sterile heavy neutrinos. The majorana contribution M_L takes the form

$$M_L = \frac{1}{2} \begin{pmatrix} \mathcal{M}_\nu & \mathcal{M}_{N\nu} \\ \mathcal{M}_{\nu N} & \mathcal{M}_N \end{pmatrix}, \tag{4.6}$$

where

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & 0 & 0 & -h_\chi^{34}\nu_\chi \\ 0 & V h_S^{11} & V h_S^{12} & h_\phi^{14}\nu_\chi \\ 0 & V h_S^{21} & V h_S^{22} & h_\phi^{24}\nu_\chi \\ h_\chi^{43}\nu_\chi & h_\phi^{14}\nu_\chi & h_\phi^{24}\nu_\chi & V h_S^{44} \end{pmatrix}.$$

The entries of the upper 3×3 submatrix correspond to majorana masses for the ordinary neutrinos of the three SM families, which are generated with the six dimensional representation of the scalar sector. If such VEV were taken as null, or if we chose discrete symmetries to forbid these terms, they can be generated through the see-saw mechanism of the form $m_D^\dagger M_R^{-1} m_D$. The other mass matrices are given by

$$\mathcal{M}_N = \begin{pmatrix} V h_S^{11} & V h_S^{12} & -2h_\phi^{14}\nu_\chi \\ V h_S^{21} & V h_S^{22} & -2h_\phi^{24}\nu_\chi \\ -2h_\phi^{14}\nu_\chi & -2h_\phi^{24}\nu_\chi & V h_S^{44} \end{pmatrix},$$

$$\mathcal{M}_{\nu N} = \begin{pmatrix} 0 & h_\rho^{11}\nu_\rho & h_\rho^{12}\nu_\rho & 0 \\ 0 & h_\rho^{21}\nu_\rho & h_\rho^{22}\nu_\rho & 0 \\ -\nu_{\eta_1} h_\eta^{43} & 0 & 0 & h_\rho^{44}\nu_\rho \end{pmatrix}, \ \mathcal{M}_{N\nu} = \begin{pmatrix} 0 & 0 & h_\eta^{34}\nu_{\eta_1} \\ -h_\rho^{11}\nu_\rho & -h_\rho^{12}\nu_\rho & 0 \\ -h_\rho^{21}\nu_\rho & -h_\rho^{22}\nu_\rho & 0 \\ 0 & 0 & -h_\rho^{44}\nu_\rho \end{pmatrix}.$$

Where we have taken into account the VEV's of the scalar triplets χ, ρ, η, the adjoint ϕ and the sextext S. The adjoint VEV's ensure the direct mixings between $\ell^{(n)}$ and the mirrors $\ell^{(4^*)}$. The Dirac terms of (4.5) are

$$m_D = \frac{1}{2}\begin{pmatrix} \nu_\rho h_{\nu_3}^{3\rho} & \nu_\rho h_{\nu_1}^{3\rho} & \nu_\rho h_{\nu_2}^{3\rho} & \nu_\rho h_{\widetilde{\nu}}^{3\rho} \\ \nu_\eta h_{\nu_3}^{1\eta} & \nu_\eta h_{\nu_1}^{1\eta} & \nu_\eta h_{\nu_2}^{1\eta} & \nu_\eta h_{\widetilde{\nu}}^{1\eta} \\ \nu_\eta h_{\nu_3}^{2\eta} & \nu_\eta h_{\nu_1}^{2\eta} & \nu_\eta h_{\nu_2}^{2\eta} & \nu_\eta h_{\widetilde{\nu}}^{2\eta} \\ \nu_\eta h_{\nu_3}^{4\eta} & \nu_\eta h_{\nu_1}^{4\eta} & \nu_\eta h_{\nu_2}^{4\eta} & \nu_\eta h_{\widetilde{\nu}}^{4\eta} \\ \nu_\chi h_{\nu_3}^{1\chi} & \nu_\chi h_{\nu_1}^{1\chi} & \nu_\chi h_{\nu_2}^{1\chi} & \nu_\chi h_{\widetilde{\nu}}^{1\chi} \\ \nu_\chi h_{\nu_3}^{2\chi} & \nu_\chi h_{\nu_1}^{2\chi} & \nu_\chi h_{\nu_2}^{2\chi} & \nu_\chi h_{\widetilde{\nu}}^{2\chi} \\ \nu_\chi h_{\nu_3}^{4\chi} & \nu_\chi h_{\nu_1}^{4\chi} & \nu_\chi h_{\nu_2}^{4\chi} & \nu_\chi h_{\widetilde{\nu}}^{4\chi} \end{pmatrix}. \tag{4.7}$$

When the quarks and leptons spectra are compared (see table 5), it is observed that they are equivalent in the sense that both introduce the same quantity of particles in the form of left-handed triplets and right handed singlets (singlet components of neutrinos are taken). Nevertheless, the Yukawa Lagrangian (and hence the mass matrices) of quarks and leptons are not equivalent because the quarks have different values of the X quantum number with respect to the leptons, this fact puts different restrictions on the terms of both Yukawa Lagrangians.

In the limit $\nu_\rho, \nu_\eta << \nu_\chi$ and $V = 0$, the Physics beyond the SM could be decoupled at low energies leaving an effective theory similar to a two Higgs doublet model (2HDM) with the fermionic fields of the SM and the right-handed neutrinos that we introduced in the particle content $\nu_{1R}, \nu_{2R}, \nu_{3R}$ to generate Dirac type masses and be able to relate the neutrino sector with the up quark sector. It allows to give a large mass to the up quark sector and the mass pattern for the neutrinos. In this limit, the mass matrices that are generated would be similar to the ansatz proposed in Ref. [9]. Considering the upper 3×3 submatrix of m_D in Eq. (4.7) and imposing discrete symmetries, it can be written in the form

$$m_D = \frac{1}{2}\begin{pmatrix} \nu_\rho h_{\nu_3}^{3\rho} & \nu_\rho h_{\nu_1}^{3\rho} & 0 \\ \nu_\eta h_{\nu_3}^{1\eta} & \nu_\eta h_{\nu_1}^{1\eta} & \nu_\eta h_{\nu_2}^{1\eta} \\ 0 & \nu_\eta h_{\nu_1}^{2\eta} & \nu_\eta h_{\nu_2}^{2\eta} \end{pmatrix}. \tag{4.8}$$

considering the same Yukawa couplings within each generation (i.e. the same $h_{\nu_m}^{n\Phi}$ for each pair $n\Phi$), we can write the matrix (4.8) as

$$m_D = \frac{\nu_\eta}{\sqrt{2}}\begin{pmatrix} ct_\beta & ct_\beta & 0 \\ \delta b & b & b \\ 0 & a & a \end{pmatrix}, \tag{4.9}$$

where $t_\beta \equiv \tan\beta = \frac{\nu_\rho}{\nu_\eta}$ is a scalar mixing angle and δ is a real parameter that is fitted in agreement with the neutrino oscillation data. If the third generation is ν_3, the second is ν_1 and the first is ν_2, and taking $M_R = M_{diag}(\epsilon_{M3}, \epsilon_{M2}, \epsilon_{M1})$, we obtain the same mass ansatz and mixing as the Ref. [9]. Thus, from the see-saw mechanism we get

$$m_\nu = -m_D^\dagger M_R^{-1} m_D = m_\nu^0 \begin{pmatrix} \delta^2\overline{\epsilon}+\omega & \delta\overline{\epsilon}+\omega & \delta\overline{\epsilon} \\ \delta\overline{\epsilon}+\omega & \epsilon+\omega & \epsilon \\ \delta\overline{\epsilon} & \epsilon & \epsilon \end{pmatrix}, \tag{4.10}$$

with $m_\nu^0 = \frac{\nu_\eta^2}{2M}$, $\epsilon = \frac{a^2}{\epsilon_{M_1}} + \frac{b^2}{\epsilon_{M_2}}$, $\overline{\epsilon} = \frac{b^2}{\epsilon_{M_2}}$, $\omega = \frac{c^2 t_\beta^2}{\epsilon_{M_3}}$, $\tan 2\theta_{23} \sim \frac{2r\omega}{\epsilon(\delta^2 - r)}$, $\tan 2\theta_{12} \sim \frac{2g}{f}$, $\theta_{13} \sim \frac{\epsilon(\delta+r)}{2^{3/2} r\omega}$, $m_1 \sim \epsilon m_\nu^0 \left\{1 - g\sin 2\theta_{12} + f\sin^2\theta_{12}\right\}$, $m_2 \sim \epsilon m_\nu^0 \left\{1 + g\sin 2\theta_{12} + f\cos^2\theta_{12}\right\}$, $m_3 \sim 2\omega m_\nu^0$, $r = \frac{\epsilon}{\overline{\epsilon}}$, $g = \frac{|r-\delta|}{\sqrt{2r}}$, and $f = \frac{\delta^2 - 2\delta - r}{2r}$. As it is discussed in Ref. [9], if $m_3 \sim \sqrt{\Delta m_{atm}^2}$, $m_2 \sim \sqrt{\Delta m_{sol}^2}$, and taking $t_\beta = \frac{\nu_\rho}{\nu_\eta} \gg O(1)$, it is possible to obtain a natural fit for the observed neutrino hierarchical masses and mixing angles. This result shows the good behavior of the model.

4.3 The mixing between the bottom quark and its mirror

In order to look for a solution to the deviation from the b asymmetries, let us assume that the exotic quarks with charge 1/3 acquire their mass in the first SSB and that they are basically decoupled at electroweak energies. On the other hand, let us suppose that the mass matrix of the three generations of down quarks is approximately diagonal. In this way, the mixing between the down quark of the third generation (b quark) and its corresponding mirror can be written as (see Eq. 4.3)

$$\begin{pmatrix} \bar{d}_2 & \overline{\widetilde{d}} \end{pmatrix}_L M \begin{pmatrix} d_2 \\ \widetilde{d} \end{pmatrix}_R ,$$

$$M \equiv \begin{pmatrix} h_{d_2}^{2\rho}\nu_\rho & h_\phi^{24}\nu_\chi \\ 0 & Y_{\widetilde{d}}^{4\rho}\nu_\rho \end{pmatrix} \tag{4.11}$$

The eigenvalues of this mass matrix M, that correspond to the masses of the b-quark and the mirror fermion are $h_{d_2}^{2\rho}\nu_\rho$ and $Y_{\widetilde{d}}^{4\rho}\nu_\rho$, respectively. To diagonalize the mass matrix the following rotation is proposed

$$\begin{pmatrix} b \\ \widetilde{b} \end{pmatrix}_{L(R)} = V_{L(R)}^\dagger \begin{pmatrix} d_2 \\ \widetilde{d} \end{pmatrix}_{L(R)} \tag{4.12}$$

where b and $\widetilde{b}$ are the mass eigenstates for the bottom quark and its mirror fermion respectively. V_L and V_R are 2×2 matrices of rotation obtained from the matrices $MM^\dagger$ and $M^\dagger M$, respectively (see Eq. 4.11). We shall assume that the rotation angle of the left-handed quarks (θ_L) is small enough, since it would be tightly restricted by the electroweak processes. For the right-handed angle we get

$$\tan 2\theta_R = \frac{2h_\phi^{24}\nu_\chi Y_{\widetilde{d}}^{4\rho}\nu_\rho}{(Y_{\widetilde{d}}^{4\rho}\nu_\rho)^2 - (h_{d_2}^{2\rho}\nu_\rho)^2 - (h_\phi^{24}\nu_\chi)^2} \approx \frac{2M_{Z'}M_F}{M_F^2 - M_{Z'}^2} \tag{4.13}$$

in the last line the b quark mass was neglected and the VEV ν_χ was approximated to $M_{Z'}$.

When writing the neutral currents for d_2 and its mirror $\widetilde{d}$ we get

$$\begin{aligned} \mathcal{L}_b^{NC} &= \frac{g}{2C_W}\overline{d_2}\gamma_\mu\left[\left(1 - \frac{2}{3}S_W^2\right)P_L - \frac{2}{3}S_W^2 P_R\right]Z^\mu d_2 \\ &+ \frac{g}{2C_W}\overline{\widetilde{d}}\gamma_\mu\left[\left(1 - \frac{2}{3}S_W^2\right)P_R - \frac{2}{3}S_W^2 P_L\right]Z^\mu \widetilde{d} \end{aligned} \tag{4.14}$$

After making the rotations for left and right-handed components of d_2, $\widetilde{d}$ quarks, and taking $\theta_L = 0$, we can write the right-handed current of the quark bottom mass eigenvalues as

$$\frac{g}{2C_W}\overline{b}\gamma_\mu\left(\sin^2\theta_R - \frac{2}{3}S_W^2\right)P_R Z^\mu b \tag{4.15}$$

and the electroweak right-handed coupling is modified by a factor

$$\delta g_R = \sin^2\theta_R \tag{4.16}$$

By making a combined fit for LEP and SLD measurements in terms of the left and right currents of the b quark, and substracting the central value of the SM it is obtained that [10]

$$\delta g_R = 0.02 \tag{4.17}$$

It means that in order to solve the problem of the deviation of the anomaly A_b, it is necessary for the right-handed mixing angle to be of the order of $\sin\theta_R \approx 0.1$. Replacing this value into Eq. (4.13) we find that $M_{Z'} \approx 10M_F$. This is a reasonable value if the mirror fermions lie at the electroweak scale and the first breaking of the 331 model is of the order of the TeV scale.

5 Conclusions

A general analysis of 331 models with β arbitrary under the assumption of minimal exotic spectrum, shows the possibility of finding vector-like models with mirror fermions when the criterion of cancellation of anomalies is taken. The existence of mirror fermions provides a possible source to solve the problem of the deviation of the bottom quark assymmetries from the SM predictions. On the other hand, the vector-like nature of the model gives a possible solution to the fermion mass hierarchy problem and in particular to the neutrino mass and mixing pattern. If we add the assumption that no exotic charges are present in the model, the minimal vector-like models that could contain the SM fermions are the ones with four generations and $\beta = \pm 1/\sqrt{3}$.

In this manuscript we study the version with $N = 4$ and $\beta = -1/\sqrt{3}$, which is a vector-like model consisting of 3 triplets containing the SM fermions plus one triplet containing mirror fermions of one of the SM families. We choose the mirror fermions to be associated with the third family of the SM. This $N = 4$ model is different from similar 331 versions considered in the literature, and posseses strong phenomenological motivations: the right-handed coupling of the $b-$quark with the Z_μ gauge boson could be modified and may in turn explain the deviation of the b asymmetries with respect to the SM prediction. In order to solve the A_b puzzle, the right-handed mixing angle should be of the order of $\sin\theta_R \approx 0.1$, which in turn leads to $M_{Z'} \approx 10M_F$ with $M_{Z'}$ and M_F denoting the masses for the exotic neutral gauge boson and the mirror fermion respectively, this relation is reasonable if M_F lies at the electroweak scale and the breaking of the 331 model lies at the TeV scale. On the other hand, vector-like models are necessary to explain the family hierarchy. From the phenomenological point of view, the model provides the possibility of generating ansatz for masses at low energies in the quark and lepton sector. It worths saying that the Physics beyond the SM could be decoupled at low energies leaving an effective theory of two Higgs doublets with right-handed neutrinos, and that the mass matrices generated are similar to the ansatz proposed by Ref. [9]. From such ansatz, a natural fit for the neutrino hierarchical masses and mixing angles can be obtained.

Finally, we could find other possible 331 vector-like versions with mirror fermions. For instance, we can analyze the model with $N = 4$ but with the mirror fermion associated with another SM family. Moreover, several vector-like models with $N \geq 4$, with more mirror fermions could be studied from phenomenological grounds (see table 2). In particular, we observe from table 2 that $N = 6$ contains models that are vector-like with respect to $SU(3)_L$ in the quark and lepton sectors.

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References

[1] Pisano, F. and V. Pleitez, Phys. Rev. **D46**, 410 (1992); P.H. Frampton, Phys. Rev. Lett. **69**, 2889 (1992); R. Foot, O.F. Hernandez, F. Pisano and V. Pleitez, Phys. Rev. **D47**, 4158 (1993); P. H. Frampton, P. Krastev, J. T. Liu, Mod. Phys. Lett. **A9**, 761 (1994); D. Ng, Phys. Rev. **D49**, 4805 (1994); L.A. Sánchez, W.A. Ponce, and R. Martínez, Phys. Rev. **D64**, 075013 (2001); Nguyen Tuan Anh, Nguyen

Anh Ky, Hoang Ngoc Long, Int. J. Mod. Phys. **A16**, 541 (2001); W.A. Ponce, J.B. Flórez and L.A. Sánchez, Int. J. Mod. Phys. **A17**, 643 (2002); W.A. Ponce, Y. Giraldo, and L.A. Sánchez, Phys. Rev. **D67**, 075001 (2003).

[2] E. D. Froggatt and H. B. Nielsen, Nucl. Phys. B147, 277 (1979); J. Schechter and J. W. F. Valle, Phys. Rev. **D22**, 2227 (1980); S. M. Barr, Phys. Rev. **D24**, 18951 (1981); S. M. Barr. Phys. Rev. Lett. 92, 101601 (1994); Yosef Nir, Yael Shadmi, JHEP 0411 (2004) 055.

[3] G. Gonzalez-Sprinberg, R. Martinez, O. A. Sampayo, Phys. Rev. **D71**, 115003 (2005).

[4] M. S. Chanowitz, Phys. Rev. Lett. 87, 231802 (2001); *ibid.* Phys. Rev. **D60**, 073002 (2002).

[5] Rodolfo A. Diaz, R. Martinez, F. Ochoa [arXiv: hep-ph/0411263]; to appear in Phys. Rev. D.

[6] Rodolfo A. Diaz, R. Martinez, F. Ochoa, Phys. Rev. **D69**, 095009 (2004); Rodolfo A. Diaz, R. Martinez, J. Mira, J.- Alexis Rodriguez, Phys. Lett. **B552**, 287 (2003); R. Martinez, N. Poveda, J.-Alexis Rodriguez [arXiv:hep-ph/0307147].

[7] R. Foot, H.N. Long and T.A. Tran, Phys. Rev. **D50**, R34 (1994); H.N. Long, Phys. Rev. **D53**, 437 (1996); *ibid*, **D54**, 4691 (1996); H.N. Long, Mod. Phys. Lett. **A13**, 1865 (1998).

[8] P. Bomert, C.P. Burgess, J.M. CLine, D. London and E. Nardi, Phys. Rev. D**54**, 4275 (1996); G. Altarelli, F. Caravaglios, G.F. Giudice, P. Gambino and G. Ridolfi, JHEP **06**, 018 (2001); X. He and G. Valencia, Phys. Rev. D **66**, 013004 (2002) and Phys. Rev. D**68**, 033011 (2003).

[9] David Atwood, Shaouly Bar-Shalom and Amarjit Soni, [arXiv: hep-ph/0502234].

[10] J. Dress, [arXiv: hep-ex/0110077]; D. Abbaneo *et. al.*, [arXiv: hep-ex/0112021]; Xiao-Gang He and G. Valencia, Phys.Rev. D**66** (2002) 013004; Erratum-ibid. D**66** (2002) 079901.